# Outdoor beam tracing over undulating terrain

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Constructing noise maps for large areas is a tedious and CPU intensive task, even if engineering approximations to outdoor sound propagation models are introduced. This paper reports on an effort to optimize beam tracing in order to reduce the number of generated paths between source and observer, while still guaranteeing the required accuracy.

Beam tracing is a ray based technique that groups rays into spatial coherent beams. These beams are restricted to the geometry at hand. Due to its nature it is very efficient in highly occluded areas. However, the proposed modifications allow the algorithm to be used efficiently in sparse outdoor scenes with complex undulating terrain as well. The beam tracing algorithm gradually turns into a Monte Carlo ray tracing algorithm while it propagates through the scene.

Beams are assigned a chance of survival depending on the distance traveled. To conserve energy the remaining beams are assigned a weight that grows with propagation distance. Statistically the algorithm redistributes energy perfectly but Monte Carlo noise is still visible if no counter measures are taken. Therefore super-sampling is used which suits the beam tracing algorithm well. Another technique that reduces the variance on the noise significantly is to distribute the energy of highly weighted beams to receivers in the neighborhood. Efficient data structures are needed to support these operations.

The improved beam tracing algorithm has been implemented and tested in the mountainous region of the Brennerpass (Austria) with good results. The calculation time of the simulation can easily be traded with the variance and resolution of the results. Noise mapping and ray tracing algorithms

## 1 Introduction

Environmental noise caused by traffic, industrial activities and recreation forms a increasing threat to public health. Policy makers try to turn the tide by enacting more strict regulations. Such regulations require reliable tools to analyse or predict the impact of noise sources on the environment. In case of traffic noise, computational methods to construct noise maps of large areas are tedious and computational complex, even when engineering approximations are applied. This paper reports on an effort to optimize beam tracing in order to reduce the number of generated paths between source and observer, while still guaranteeing the required accuracy.

## 2 Portal based beam tracing

### 2.1 Principle

Beam tracing [1, 2, 3, 4] is a technique based on ray tracing. Where in ray tracing sound waves are treated as high frequent and are represented by series of straight lines, the beam tracing approach exploits the spatial coherence of rays and bundles them in beams. From the source, beams are emitted as a set of an infinite number of rays sharing the same source. These beams are followed through the polyhedral geometrical scene until a receiver is hit. Subsequently, a piecewise linear path between source and receiver is constructed and the acoustical contribution of the path is accumulated in the receiver.

Beams are kept polyhedral and convex to allow an easy representation and detection of receivers. A beam $\mathcal{B}$ can be represented as an intersection of half-spaces $\Sigma_i$. $\mathbf{n}_i$ and $d_i$ are the normal vector and signed distance to the origin of the boundary plane of $\Sigma_i$.

$$\mathcal{B} = \bigcap_i \Sigma_i \tag{1a}$$

$$\mathbf{p} \in \Sigma_i \Leftrightarrow \mathbf{n}_i \cdot \mathbf{p} + d_i > 0 \tag{1b}$$

To trace the beams through the scene, a spatial subdivision of the scene is used. The scene is represented by a convex cell complex consisting of linked polyhedral cells. The construction of the 3D model starts from plan view where building edges and height lines from the terrain are triangulated using a 2D constrained Delaunay triangulation. The triangles of the plan view are merged into convex polygons and erected to form the polyhedral cells. The union of this set of cells is then a 3D model of the non-obstructed space of the scene. Boundary polygons are fully shared by two cells or not shared at all. These boundary polygons are *portals* through which the beams can travel from one cell to another, or back into the originating cell (reflection is considered to be transmission after transformation). A winged-pair adjacency graph [5] is constructed representing the neighbor relationships between the cells. This allows to travel through the scene efficiently.

The propagation algorithm starts by emitting for each source an omni directional beam in the cell containing the source. New beams are constructed from each boundary polygon of the starting cell. A Sutherland-Hodgeman clipping algorithm limits these beams to only include the rays transmitted through this portal. For each new beam, this step is repeated recursively and depth-first, until some stopping criterium is reached.

Diffraction is added by using the Uniform Theory of Diffraction (UDT) [6, 7, 8]. A beam incident on a wedge has a diffracted beam that bounces of the wedge omni directionally. This beam however is no longer polyhedral and has a line segment as source rather than a single point. This is taken care of by tracing a conservative polyhedral approximation of the diffracted beam. During the path generation phase, additional checks on the generated paths verify their validity. Visibility of the diffracted beams is limited to the shadow region because the relative contribution of diffraction in directly lit regions is small.

Polyhedral beam tracing limits beams to the geometry and makes no approximation of the available polyhedral bounding geometry. The result is that beam tracing guarantees to find for all source and receiver pairs all existing paths under a given number of reflections and diffractions.

To obtain equally accurate results with pure ray tracing, it is necessary to emit a much higher number of rays in both horizontal and vertical direction. Consequently, in outdoor situations, a high number of rays will hit the sky without contributing to any receiver. Beam-tracing does not suffer from this problem, because the beams stretch from top to bottom anyway.

## 2.2 Limitations

Due to its nature, portal based beam tracing is known to be very easy and efficient for densely occluded environments like indoor architectural scenes and street canyons.

When the algorithm is applied to sparsely occluded environment over an undulating ground surface, a substantial number of portals extent from the bottom to top of the scene, only to model the terrain. The ratio of transparent to non-transparent portals increases tremendously, causing unnecessary clipping and fragmenting of the beams, whereas the number of paths remains low. The beam tracer becomes a major bottleneck of the noise mapping process.

## 3 Russian roulette

The Russian roulette method is related to the Monte Carlo technique, and is used to stochastically select or terminate paths without introducing bias [9, 10].

To overcome the performance bottleneck of the beam tracer over undulating terrain, a Russian roulette is applied to the beam tracer as follows. At depth $N$ of the recursive algorithm, before tracing new beams generated from one parent beam, each of the new beams is subjected to the roulette. In this roulette, a random number $x$ is drawn from the uniform distribution in $[0, 1)$ and compared to a fixed rejection ratio $\rho_N$. If $x < \rho_N$, the new beam is rejected. Otherwise the beam survives and recursion continues.

$$P\left[\text{survival}\right] = P[x > \rho_N] = 1 - \rho_N \qquad (2)$$

This simple addition to the beam tracing algorithm results in a huge speedup of the beam tracer, as only a fraction of the original number of beams will survive the roulette. With fixed rejection ratio $\rho$, only an average fraction $(1 - \rho)^N$ of the beams remain at recursion depth $N$.

If the above scheme is applied as is, it is easily seen that the conservation of energy is violated, as the energy in the rejected beams is simply ignored. As a result of that, the overall level in the noise map will be underestimated.

This can be compensated by assigning the energy of the rejected beams to the surviving ones. Therefore a path weigth $W_N$ to each surviving beam is added, which equals to its parent's path weight $W_{N-1}$ multiplied by $\frac{1}{1-\rho_N}$.

$$W_0 = 1 \qquad (3a)$$

$$W_N = \prod_{i=1}^{N} \frac{1}{1 - \rho_i} \qquad (3b)$$

With this path weight, on average, the conservation of energy is respected. This can be validated as follows. Consider the correct contribution $P$ of a single path between source and receiver. When the Russian roulette is applied, the actual contribution $P_k$ will either be zero or $\frac{1}{1-\rho}P$. If actual contributions of that path of independent simulations are averaged, they will average to the correct value $P$ ($s_k$ is 0 if the beam is rejected, and 1 if it survived).

$$E\left[P_k\right] = E\left[\frac{s_k}{1 - \rho}P\right] = \frac{E[s_k]}{1 - \rho}P = P \qquad (4)$$

## 4 Adaptive noise reduction

Though the solution described above will give a result that is correct on average, it will contain a lot of high frequency noise due to the Russian roulette. This is typical to Monte Carlo techniques. Energy previously distributed over many different beams is now concentrated in only a few. This is reflected by extremely large path weights. With a fixed rejection ratio $\rho = 10\%$ and at recursion

depth $N = 200$ (which is not uncommon for large outdoor scenes), pathweights of $W_{200} = 1.4 \cdot 10^9 = 92dB$ are found! Consequently, large local errors can be expected.

Though the human eye itself is capable of spatially filtering the result, or a spatial box filter can be applied, an adaptive noise reduction technique that performs better than any post processing filter will be described. The need for an adaptive noise filter is easily understood. Paths with weight 1, which have not passed the roulette, do not need to be filtered. Paths with a higher weight contain more energy than they should and this needs to be spatially filtered. A filter that adapts to the path weight is capable of preserving the detail created by paths with a small weight while reducing the high energy noise caused by paths with a high weight. Once the unfiltered noise map is built, no information about the individual contributions and their path weights is available anymore for the post processing.

Instead of using the path weight $W$ to scale the contribution $P$ and assign it to one receiver, the unscaled $P$ is distributed to $W$ neighbor receivers. The neighbors chosen are the ones most closely to the actual receiver.

When the receivers lay on a uniform grid, the selection of the nearest neighbors is easy and efficient. In our case receivers are placed on a quasi uniform grid where around buildings and objects of interest denser grids are used. Therefor we store the receiver positions in a Kd-Tree. Construction of the Kd-Tree is performed once before the beam tracing starts and takes $O(N \cdot \log(N))$ with N the number of receivers. Radial range search within a Kd-Tree has an expected execution time of $O(\log(N) + k)$ with $k$ the number of points returned by the search. For a radial search on a quasi uniform grid $k$ is proportional to the squared radius. If the search does not produce enough neighbors, the search can be reissued with a larger radius.

# 5 Applications and results

The improved beam tracing algorithm as described above has been implemented and tested in the mountainous region of the Brennerpass (Austria) with good results, Figure 1. The region picked surrounds the village *Steinach am Brenner* and its bounding box in UTM coordinates (meters) is:

$$[685463, 688576] \times [5218064, 5220159]$$

## 5.1 Variance Statistics

The variance at a location in the noise map is defined as the variance over the series of results obtained by consecutive runs of the simulation. The distribution of variance
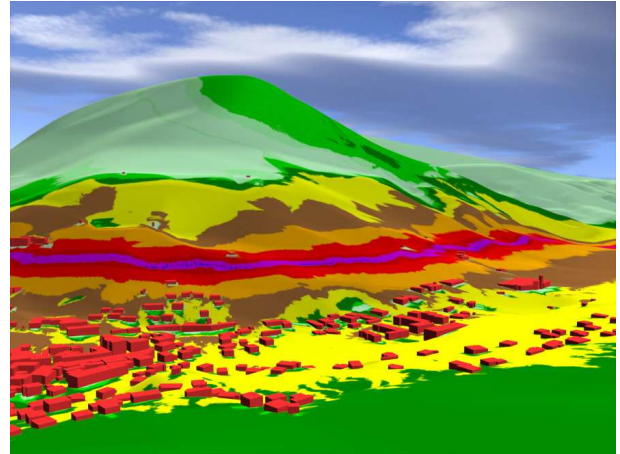


Figure 1: A 3D view of the simulated area, *Steinach am Brenner* (Brennerpass, Austria)

over the map measures how well consecutive simulations of the same region with the same parameter set will correspond. The only difference between those consecutive simulations being the seed of the random generator. The 90 and 95 percentile measure how well the largest differences behave under the different simulation conditions. The 50 percentile is a measure for the average spatial variance observable in consecutive simulations. Figure 2 shows the 50, 90 and 95 percentile of the variance over the computed map. In the experiment about 20 simulations were performed on average.

The variance decreases drastically with the rejection ratio. Adaptive noise filtering reduces the variances on the levels by almost a decade for the 90 and 95 percentile, the effect on the 50 percentile is less pronounced. It are however the high variances that needs the most reduction as they skew the results and produce large artifacts in the noise maps. Figure 3 shows a noise map with and without the adaptive noise filter.
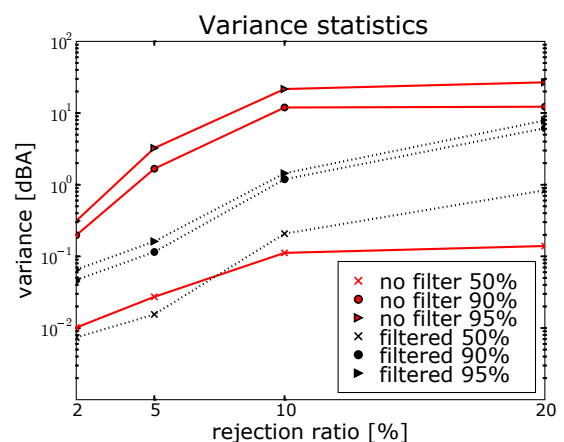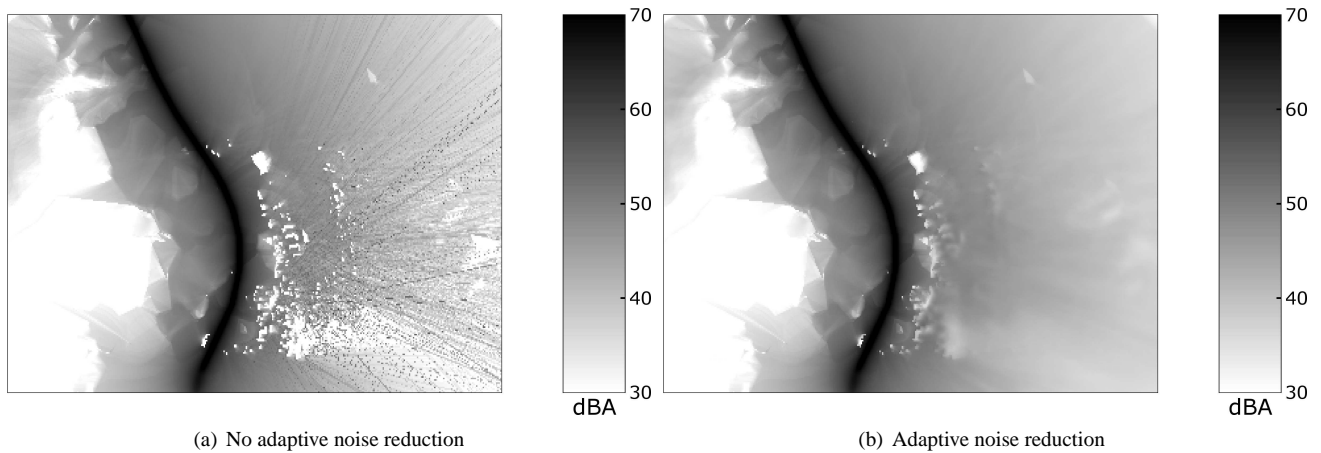


Figure 2: Variance as function of rejection ratio

(a) No adaptive noise reduction



(b) Adaptive noise reduction

Figure 3: Effect of adaptive noise reduction filter on the resulting noise map. Grid resolution is 10 m, rejection ratio is 10%.



(a) No adaptive noise reduction


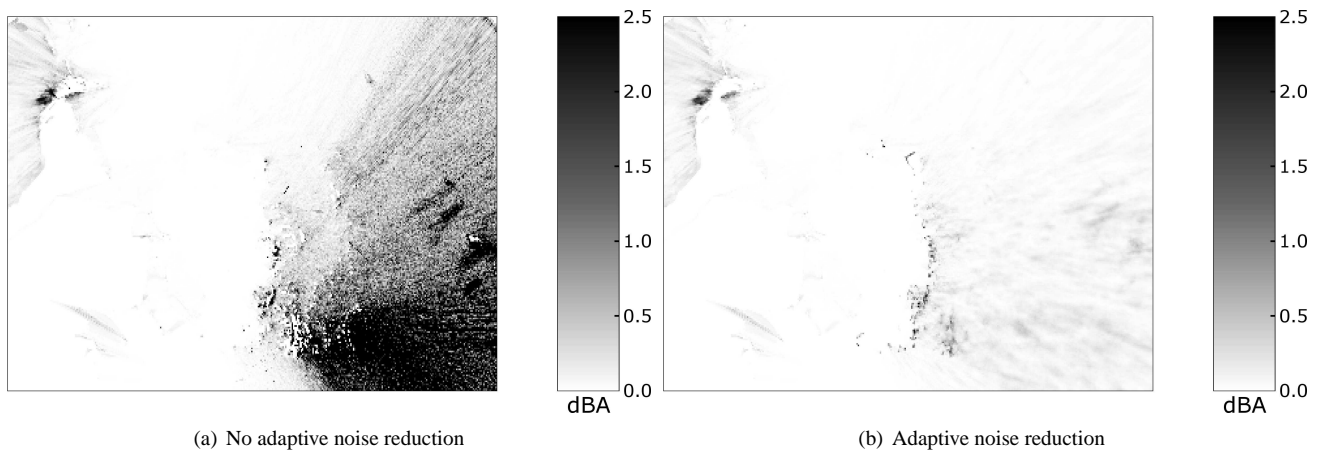
(b) Adaptive noise reduction

Figure 4: Effect of adaptive noise reduction filter on variance. Grid resolution is 10 m, rejection ratio is 10%.

Figure 4 shows the spatial distribution of the variance. The dark area in figure 4(a) is caused by the presence of houses which trigger the roulette. The houses induce a refined meshing locally which causes the beams to fragment. The number of portals encountered for propagation over a given distance increases. Therefore the probability of rejection increases when the beam tracer passes through a densely meshed region. Figure 4(b) shows the effect of the adaptive noise filter on the variance.

## 5.2 Performance

Figure 5 shows the performance of the algorithm for different grid resolutions. It is obvious that the total time needed for the simulation increases for increasingly dense grids. These graphs however reveal how well the algorithm performs when the number of receivers per beam changes. When the resolution is set too coarse (100m for instance) the beams propagate too long before they encounter a receiver and can eventually decide to

stop further beam tracing based on the acoustic energy present in the beams. When the resolution is set too fine (2m in our case) the roulette does not have enough opportunity to cancel beams. Figure 5 reveals that a resolution of 5m is the most efficient in terms of time needed per receiver, as well that the simulation at 50m runs faster than the one at 100m spacing. To obtain results at a 2m resolution it is also better to repeat the simulation with 5m resolution 4 times with an appropriate offset.

Figure 6 shows the amount of variance that is generated per time step. The rejection ratio which reports the lowest value is the most efficient in reducing the variance in the noise map. The selected variance measure was the 95 percentile of the total variance distribution. The performance shows a maximum for a rejection ratio of 5%. The occurence of maximum is caused by the rapid growth in computation time for lower rejection ratios although they decrease the variation only marginally.
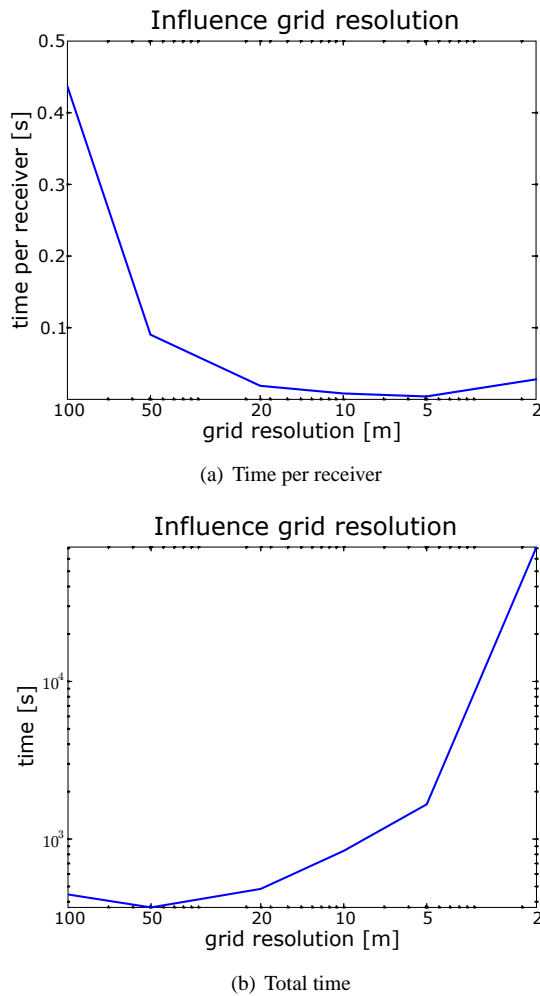
## Influence grid resolution



(a) Time per receiver

## Influence grid resolution



(b) Total time

Figure 5: Performance of beam tracing for different grid resolutions (rejection ratio = 10%)
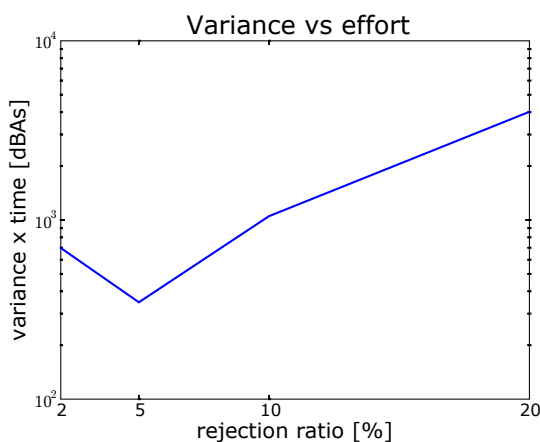
## Variance vs effort



Figure 6: Variance performance as function of rejection ratio

# 6   Possible improvements

- The Russian roulette is applied independently of the source directivity. It is possible to reduce the variance of the solution by assigning a higher survival probability to beams with higher source contribution.

- In the adaptive noise filter, the path contribution $P$ is uniformly applied to all neighbors in the range. Better results may be achieved by applying a different distribution such as a Gaussian bell.

- In this paper a general Russian roulette was implemented while the maximum number of reflection and diffraction events was still limited to a fixed number. This too introduces a bias to the result which can be avoided by applying a secondary Russian roulette to this stopping criterium.

# 7   Conclusion

Beam tracing of large outdoor areas with undulating terrain suffers from fragmented beams. An improvement of the beam tracing algorithm is proposed that uses a Russian roulette to eliminate beams from the simulation. The loss of energy caused by the random elimination is compensated by assigning the lost energy to the remaining beams. The spatial noise observed by this Monte Carlo like redistribution is reduced by using an adaptive noise filter. This filter redistributes the energy in the vicinity of the beams. The key advantage over post-processing is that in areas where the Russian roulette is not active (e.g. close to the source), the sound levels remain unchanged.

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