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THE EFFECT OF DIFFUSE REFLECTIONS ON NOISE MAPS USING PHONON MAPPING

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Abstract

Constructing noise maps for large urban areas is a tedious and CPU intensive task. Engineering approximations to outdoor sound propagation are commonly introduced to reduce the computational complexity. In particular, diffuse reflections are often neglected because of their great computational expense. This paper reports on an effort to approximate the diffusely reflected field by using a technique called phonon mapping.

In noise mapping, many techniques can and are adapted from the field of computer graphics where various techniques to approximate indirect lighting exist. A popular technique in computer graphics is photon mapping. It was introduced in 1995 by Jensen and is known as one of the fastest algorithms for simulating indirect lighting. In this paper, it will be shown how this photon mapping algorithm can be adapted to the field of noise mapping to simulate diffuse reflections.

The phonon mapping algorithm exists of two passes. In the first pass, a phonon map is build by bouncing phonons, abstract sound particles, through the environment. On each interaction with a (partially) diffusely reflecting surface, it is stored in the phonon map. The Russian roulette technique is used to determine if the phonon is absorbed, specular or diffusely reflected. Upon reflection, the phonon is traced through the environment again. This process is continued until the phonon map is sufficiently populated. The second pass is a more traditional backward ray tracer. Rays are emitted from the receivers. Specular reflections are handled as usual but for diffuse reflections, no secondary, reflected rays are cast. Instead, the phonon map is queried for nearby phonons. Their presence is used to estimate the energy that is available from this diffuse reflector.

DEFINITIONS

Directional sound power $\mathcal{D}$: sound power per unit solid angle $d\omega$. Its units are $\frac{\mathrm{W}}{\mathrm{sr}}$.

Acoustic radiance $\mathcal{L}$: sound power per unit projected area $dA^{\perp}$ and per unit solid angle $d\omega$, with $dA^{\perp} = dA\cos\theta$ the projected area of dA on the hypothecial surface orthogonal to $d\omega$. Its units are $\frac{\mathrm{W}}{\mathrm{m^2 sr}}$.

INTRODUCTION

Computational methods to construct noise maps of large areas are tedious and computational complex, even when engineering methods are applied. Classic ray/beam tracers [3, 4, 6] often only implement specular reflections using the image sources. Diffuse reflections are often neglected because of their great computational cost and complexity, despite the fact that the diffusely reflected field can be a significant contribution to urban soundscapes. This paper reports on an effort to approximate the diffusely reflected field by using a technique called *phonon mapping*.

Phonon mapping belongs to the family of geometrical acoustics, together with methods like ray tracing and beam tracing, making the same high frequency approximation. Phonon mapping is a particle tracing technique in which packets of acoustic power called *phonons* are bounced through the geometrical scene. On diffuse surfaces, incoming phonons are stored in a *phonon map*. Later, this phonon map is used to estimate the incoming power per area which is used to find the outgoing *acoustic radiance* $\mathcal{L}$ available from this diffuse reflector.

This technique is adopted from the field of computer graphics where it is known as *photon mapping*. The latter is introduced in 1995 by Jensen [8, 9] as an efficient alternative to *radiosity* [5] and *Monte Carlo ray tracing* [10] for simulating *global illumination*.

The goal of noise mapping is to determine how much acoustic power W is reaching a receiver located at $\mathbf{r}$. It is an integral of the incoming directional power $\mathcal{D}$ over all directions:

$$W(\mathbf{r}) = \int_{\Omega} \mathcal{D}(\mathbf{r}, \omega)\, d\omega \tag{1}$$

In geometrical acoustics, this integral is written as the sum over all sound paths that terminate at $\mathbf{r}$:

$$W(\mathbf{r}) = \sum_{k} \mathcal{D}_k(\mathbf{r}, \omega_k) \tag{2}$$

This sum includes the set of all sound paths from a source to the receiver via any number of specular or diffuse reflections. It contains an infinite number of sound paths, therefore for practical applications an approximation of this sum must be made.

A traditional ray or beam tracer can only handle direct paths or paths with specular reflections only. The sum of all their contributions is W_s. Such a ray tracer misses all sound paths that have at least one diffuse reflection. The goal of the phonon mapper in this paper is exactly to estimate the sum W_d of the contributions of the latter. Finally, both results must be summed to get W:

$$W(\mathbf{r}) = W_s(\mathbf{r}) + W_d(\mathbf{r}) \tag{3}$$

FIRST PASS – PHONON TRACING

Emitting Phonons From A Single Source

A number of phonons is emitted from the source with power P until the phonon map is populated with N_{map} phonons. N_{emit} is the total number of phonons emitted and is not known beforehand because some phonons may never end up in the phonon map, and others may be stored several times.

All phonons emitted have the same power. This is important for two reasons: Firstly computational resources are not spent on low power phonons that would contribute only a little to the result, and secondly it will reduce the variance of the *radiance estimate* in the gathering pass. As a consequence, more phonons must be emitted in directions with higher power contribution. For omnidirectional sources, directions of phonon emission ω must be uniformly distributed over the unit sphere. This can be accomplished by transforming a sample (ξ_1, ξ_2) from the uniform distribution over $[0,1] \times [0,1)$ using (4).

$$\omega\left(\xi_1, \xi_2\right) = \begin{bmatrix} 2\sqrt{\xi_1\left(1-\xi_1\right)}\cos 2\pi\xi_2 \\ 2\sqrt{\xi_1\left(1-\xi_1\right)}\sin 2\pi\xi_2 \\ 1-2\xi_1 \end{bmatrix} \tag{4}$$

The sum of all phonon powers must equal P, so that each phonon will have power $P_{ph} = \frac{P}{N_{emit}}$. Since N_{emit} is initially unknown, this can only be assigned after the phonon tracing pass has ended.

Multiple Sources

In case of multiple sources with powers P_i, an extra random variable ξ_3 is used to select the source emitting the phonon. Each source can be selected with a probability $p_i = \frac{P_i}{P_{tot}}$ with $P_{tot} = \sum P_i$. Sources with more power will emit more phonons. The power of each phonon will be $P_{ph} = \frac{P_{tot}}{N_{emit}}$.

Surface Interaction

Sound power incident on a surface in point $\mathbf{r}$ with surface normal $\mathbf{n}$ is partially reflected back into the scene. The reflection coefficients R_s and R_d indicate the fraction of the incident sound power that is respectively specularly and diffusely reflected[1].

The sound power P carried by phonons hitting a surface must thus be reflected. One way to accomplish this is *path splitting*: several new phonons are emitted carrying the reflected power: one with the specular reflected power $R_s P$ and n phonons with diffusively reflected power $\frac{1}{n} R_d P$. Although this technique is straight forward, it has two major drawbacks: firstly it results in an exponential growing number of paths to be traced, and secondly the phonons in the phonon map will no longer have similar powers increasing the variance of the radiance estimate in the second pass.

[1] In this paper, the spectral and spatial dependency of both R_s and R_d is not explicitely mentioned in order not to overload the notation. Neither is the depenpency of R_s on the incoming angle θ. The implementation will take this into account.

Instead, another technique called *Russian roulette* [1, 11] is used to determine whether the phonon should be absorbed (discarded) or reflected. A random number ξ is drawn from the uniform distribution in $[0, 1)$. Depending on its value it is determined what should happen to the phonon:

$$\begin{cases} \xi \in [0, R_s) & \rightarrow \text{ specular reflection} \\ \xi \in [R_s, R_s + R_d) & \rightarrow \text{ diffuse reflection} \\ \xi \in [R_s + R_d, 1) & \rightarrow \text{ absorption} \end{cases} \quad (5)$$

In case of diffuse reflection, the new direction is sampled using ρ_d, the diffuse part of the BRDF[2]. If ρ_d is Lambertian, this is a constant and the new direction can be uniformly sampled from the entire hemisphere around the normal $\mathbf{n}$ using (4).

If ρ_d is sufficiently similar to a Lambertian BRDF, uniform sampling can still be used if the power of the outgoing phonon is modified according to the real ρ_d. However, the phonons will no longer have equal powers, increasing the variance of the radiance estimate in the gathering pass.

Multiple Frequencies

In this paper, the frequency dependency of source powers and reflection coefficients are not explicitely taken into account. It is however a trivial extension to compute with (1/3) octave-bands. Assume a source with a spectral power distribution given by a number of bands with center frequencies f_k and powers $P_{i,k}$ with $P_i = \sum_k P_{i,k}$.

It seems straight forward to emit phonons carrying full spectral information of the source. However, if R_s and R_d are frequency dependent, this complicates the surface interaction. Firstly the Russian roulette assumes scalar ranges in (5), and secondly the power spectrum of the phonon will have to be changed by the reflection.

A better solution is to emit single frequency phonons. Similar to selecting one of multiple sources, f is selected from the set f_k with probabilities $p_k = \frac{P_{i,k}}{P_i}$. This approach simplifies the roulette and the phonon power remains constant under reflection.

PHONON MAP

In the first pass, each time a phonon hits a surface with $R_d > 0$, the phonon is stored for building the phonon map. It is important to do this independently of the outcome of the Russian roulette, since the phonon map represents an estimation of the *incoming* power flux, not the outgoing one. Of each phonon, the following attributes are stored: its power P_{ph}, its position $\mathbf{r}_{ph}$ and its incoming direction ω_{ph}.

Once a sufficient number N_{map} of phonons are stored, the first pass is terminated and the phonon map is build. A balanced adaptive kd-tree [2, 11] is constructed from all stored phonons. This is a compact and efficient data-structure that allows to locate the M nearest phonons to a point $\mathbf{r}$ in $O(M \log N_{map})$ time on average. The storage of this tree is $O(N_{map})$

[2]BRDF (*Bidirectional Reflectance Distribution Function*) $\rho(\mathbf{r}, \omega_i, \omega_o)$: reflection coefficient as a function of position $\mathbf{r}$, incoming ω_i and outgoing angle ω_o.

and constructing it takes $O\left(N_{map}\log N_{map}\right)$, which performed only once before the second pass.

SECOND PASS – GATHERING PASS

In the second pass, $W_d\left(\mathbf{r}\right)$ is evaluated for each receiver. It is estimated by a sum of $\mathcal{D}_d$ in M random direction ω_k, choosen from a distribution with probability density function $p\left(\omega\right)$:

$$W_d\left(\mathbf{r}\right)=\int_{\Omega}\mathcal{D}_d\left(\mathbf{r},\omega\right)d\omega\approx\frac{1}{M}\sum_{k=1}^{M}\frac{\mathcal{D}_d\left(\mathbf{r},\omega_k\right)}{p\left(\omega_k\right)}\tag{6}$$

If the directions are uniformely distributed over the unit sphere using (4), then $p\left(\omega\right)=\frac{1}{4\pi}$, and (6) simplifies to:

$$W_d\left(\mathbf{r}\right)\approx\frac{4\pi}{M}\sum_{k=1}^{M}\mathcal{D}_d\left(\mathbf{r},\omega_k\right)\tag{7}$$

$\mathcal{D}_d\left(\mathbf{r},\omega_k\right)$ is computed by casting a ray $\left(\mathbf{r},\omega_k\right)$ in the scene. It intersects the scene in surface point $\mathbf{r}'$ with surface normal $\mathbf{n}'$. $\mathcal{D}_d\left(\mathbf{r},\omega_k\right)$ equals the acoustic radiance $\mathcal{L}_o\left(\mathbf{r}',-\omega_k\right)$ leaving the surface at $\mathbf{r}'$ towards the receiver. It can be evaluated as an integral over the unit hemisphere $\mho$ around $\mathbf{n}'$ of the incoming radiance $\mathcal{L}_i\left(\mathbf{r}',\omega''\right)$ and the BRDF $\rho\left(\mathbf{r}',-\omega_k,\omega''\right)$:

$$\mathcal{D}_d\left(\mathbf{r},\omega_k\right)=\mathcal{L}_o\left(\mathbf{r}',-\omega_k\right)=\int_{\mho}\rho\left(\mathbf{r}',-\omega_k,\omega''\right)\mathcal{L}_i\left(\mathbf{r}',\omega''\right)\left(\omega''\cdot\mathbf{n}\right)d\omega''\tag{8}$$

The BRDF can be split in a specular ρ_s and a diffuse part ρ_d. The specular part is only non-zero for the perfectly reflected direction, and can be evaluated by tracing the ray $\left(\mathbf{r}',\omega_k-2\omega_k\cdot\mathbf{n}'\right)$ and evaluating (9) again at the intersection.

$$\mathcal{L}_o\left(\mathbf{r}',-\omega_k\right)=R_s\mathcal{L}_i\left(\mathbf{r}',\omega_k-2\omega_k\cdot\mathbf{n}'\right)+\int_{\mho}\rho_d\left(\mathbf{r}',-\omega_k,\omega''\right)\mathcal{L}_i\left(\mathbf{r}',\omega''\right)\left(\omega''\cdot\mathbf{n}'\right)d\omega''\tag{9}$$

The diffuse term is estimated by querying the phonon map. In the neighbourhood of $\mathbf{r}'$, the n nearest phonons $\left(P_{ph},\mathbf{r}_{ph},\omega_{ph}\right)_j$ are found with $\|\mathbf{r}_{ph}-\mathbf{r}'\|\leqslant r_{max}$ and $n\leqslant n_{max}$. The phonons provide information on incoming power flux P_i, what must be converted to incoming radiance $\mathcal{L}_i$:

$$\mathcal{L}_i\left(\mathbf{r}',\omega''\right)=\frac{d^2P_i\left(\mathbf{r}',\omega''\right)}{\left(\omega''\cdot\mathbf{n}'\right)dAd\omega''}\tag{10}$$

$$\int_{\mho}\rho_d\left(\mathbf{r}',-\omega_k,\omega''\right)\mathcal{L}_i\left(\mathbf{r}',\omega''\right)\left(\omega''\cdot\mathbf{n}'\right)d\omega''\approx\frac{1}{A}\sum_{j=1}^{M}\rho_d\left(\mathbf{r}',-\omega_k,-\omega_{ph,j}\right)P_{ph,j}\tag{11}$$

In case of a Lambertian surface, this simplifies to:

$$\int_{\mho}\rho_d\left(\mathbf{r}',-\omega_k,\omega''\right)\mathcal{L}_i\left(\mathbf{r}',\omega''\right)\left(\omega''\cdot\mathbf{n}'\right)d\omega''\approx\frac{R_d}{A\pi}\sum_{j=1}^{M}P_{ph,j}\tag{12}$$

A is the area of the circular neighbourhood in which the n phonons are found. If r is the largest distance between $\mathbf{r}'$ and any of the n phonons, then $A=\pi r^2$.

Figure 1: SPL and error in receivers $\mathbf{r}\,(x, 0, z_0)$ with $z_0 = 10$ m; plane $z = 0$ with $R_d = 0.95$, $R_s = 0$; source $\mathbf{s}\,(0, 0, z_0)$ with $P = 0$ dB; $M = 225$, $n_{max} = 20$, $r_{max} = 10$ m.

Stratified Sampling

When evaluating (6) using directions ω_k generated by (4) with (ξ_1, ξ_2) directly sampled from $\Lambda = [0, 1] \times [0, 1)$, the spreading of the directions over the full sphere isn't as good as one may think. The samples will tend to cluster causing some directions to have many samples, while others are undersampled.

It is possible to counteract this clustering and reducing the variance on the result by using *stratified sampling* [11]. The idea is to split the sampling space Λ into $M_1 \times M_2$ equal *strata* Λ_{jk} (13), and to take each sample $(\xi_1, \xi_2)_{jk}$ from a uniform distribution over Λ_{jk}. That way, the samples are much more evenly spread over the sampling space so that no directions are undersampled.

$$\Lambda_{jk} = \left[\frac{j}{M_1}, \frac{j+1}{M_1}\right) \times \left[\frac{k}{M_2}, \frac{k+1}{M_2}\right) \tag{13}$$

APPLICATIONS AND RESULTS

Validation

A source with power $P = 0$ dB is positioned in $\mathbf{s}\,(0, 0, z_0)$ with $z_0 = 10$ m, above a very large diffuse reflector with $R_d = 0.95$ and $R_s = 0$. Using the phonon mapper, integral (6) is evaluated for receivers in positions $\mathbf{r}\,(x, 0, z_0)$. As a reference solution, (7) is evaluated with a direct solution for $\mathcal{D}_d\,(\mathbf{r}, \omega_k)$ with $M = 80000$:

$$\mathcal{D}_d\,(\mathbf{r}, \omega_k) = \mathcal{L}_o\left(\mathbf{r}', -\omega_k\right) = \frac{R_s}{\pi} \frac{P}{\|\mathbf{s} - \mathbf{r}'\|^2} \frac{(\mathbf{s} - \mathbf{r}') \cdot \mathbf{n}'}{\|\mathbf{s} - \mathbf{r}'\|} \tag{14}$$

Figure 1 shows the sound level and error for two photon mapping simulations and the reference. It can be seen that the photon mapper slightly overestimates the levels. This is due to an understimation of A in (11). A is too thightly fit around the phonons.

Variance Statistics

Monte Carlo simulations often suffer from noise or variance on the result. For the phonon mapper as described in this paper, this is mainly influenced by the number M of rays casted in the gathering pass.

The local expected error in each point of the map is estimated as the standard deviation on the results of different runs of the same simulation with different random seeds, as the average will converge to the exact solution. Figure 2 shows the result of one run and the local expected error for $M = 100$. It is clearly seen that the largest errors are expected where the source is screened by an obstacle, where not much phonons are found. Fortunately, the sound level significantely drops in these areas.

(a) diffuse component W_d of noise map

(b) local expected error on W_d

Figure 2: Diffuse component W_d of noise map and local expected error for one source $\mathbf{s}\,(30, 30, 4)$ with $P = 0$ dB, $R_d = 0.05$, $R_s = 0$, $N_{map} = 40000$, $M = 100$, $n_{max} = 20$, $r_{max} = 10$ m, receivers $\mathbf{r}\,(x, y, 4)$ on grid with $\Delta x = \Delta y = 1$ m.

Figure 3: percentiles of expected error distribution for various M (number of rays casted in gather pass), stratified vs. unstratified sampling or rays. Other parameters as figure 2.

To get an idea of the overall expected error, the distribution of the local error over the map is examined. The 90 and 95 percentiles measure how well the largest differences behave, while the 50 percentile is a measure for the average spatial error. Figure 3 shows the 50, 90 and 95 percentiles. It is clearly seen that the expected error decreases with M, however at a linearly increasing cost. It can also be seen that stratified sampling decreases the expected error for the same M, and thus without introducing any extra cost.

POSSIBLE IMPROVEMENTS

In this paper, diffraction of sound paths is not taken into account for simplicity. An polygonal beam tracer [3, 4] can easily add this to W_s for paths without diffuse reflection. However, the phonon mapper must be extended to take care of paths with both diffraction and diffuse reflection. Based on Fresnel zones, a Russian roulette may be applied to subject phonons to diffraction.

CONCLUSIONS

In this paper, it is shown how the *photon mapping* technique can be adopted to the field of noise mapping to simulate diffuse reflections. The new *phonon mapping* technique must be used in parallel to a traditional ray/beam tracer that handles the strict specular paths.

The phonon mapper consists of two passes: a phonon tracing map where phonons are distributed over the geometrical scene to form a phonon map, and a gathering pass where for each receiver the incoming diffusive field is reconstructed from the phonon map.

The variance on the result is mainly influenced by the number of gather rays. Stratified sampling can reduce the expected error without extra costs.

REFERENCES

[1] J. Arvo, D. Kirk, “Particle Transport and Image Synthesis”, Computer Graphics, **24**, 63–66 (1990)

[2] J. L. Bentley, “Multidimensional Binary Search Trees Used for Associative Searching”. Comm. of the ACM **18** (9), 509–517 (1975)

[3] B. de Greve, “3D polygonal beam tracer for acoustic simulations” (in Dutch), Master thesis, Ghent University, Belgium (2003)

[4] T. De Muer, “Policy Supporting Tools for Urban Noise Assessment”, PhD thesis, Ghent University, Belgium (2005)

[5] C. M. Goral, K. E. Torrance, D. P. Greenberg, B. Battaile, “Modeling the interaction of light between diffuse surfaces”, Computer Graphics, **18**, 213–232 (1984)

[6] P. S. Heckbert, P. Hanrahan. “Beam tracing polygonal objects”. Computer Graphics, **18**, 119–127 (1984)

[7] P. S. Heckbert, “Adaptive Radiosity Textures for Bidirectional Ray Tracing”, Computer Graphics, **24**, 145–154 (1990)

[8] H. W. Jensen, N. J. Christensen, “Photon Maps in Bidirectional Monte Carlo Ray Tracing of Complex Objects”, Computers & Graphics, **19** (2), 215–224 (1995)

[9] H. W. Jensen, “Global Illumination using Photon Maps”, Rendering Techniques ‘96, Eds. X. Pueyo and P. Schrder, Springer-Verlag, 21–30 (1996)

[10] J. T. Kajiya, “The Rendering Equation”, Computer Graphics, **20**, 143–149 (1986)

[11] M. Pharr, G. Humphreys, *Physically Based Rendering: From Theory to Implementation*. (Morgan Kaufmann, San Fransisco, ISBN 0–1255–3180–X, 2004)